A METHOD OF CALCULATING AN INDUCTION FLOWMETER FOR PIPELINES OF LARGE THROUGH CROSS SECTION

A. S. Petrazhitskii and V. É. Tsirkunov

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A two-dimensional approximation is used in models of flowmeters which operate in a wide range of magnetic Reynolds numbers Re_m. The correct formulation of the mathematical model of a flowmeter for arbitrary Re_m is given in [1]. In the case when the external magnetic field is approximated by a uniform field over some section and by an exponentially declining field along the edges of the section an analytical expression is presented for the induced magnetic field, obtained through a Fourier transform with respect to the coordinate along the channel. Calculations by the expression presented are difficult in the neighborhoods of a number of values of Re_m because of a loss of accuracy.

A general solution to the problem of the distribution of the magnetic field and potential for an arbitrary dependence of the external field on the coordinate along the channel was obtained in [2] using a representation of the magnetic field in the form of a trigonometric series with respect to the coordinate across the channel. The presence of conducting walls of the channel was approximately taken into account on the basis of a representation of the wall in the form of an infinitely thin layer with a certain resistance per unit length. The particular case of the approximation of the external magnetic field by several exponential functions was also analyzed. It should be noted that the direct use of the general solution found in [2] in a calculation of the electromagnetic characteristics of a channel is difficult when the external magnetic field is determined experimentally at a set of evenly spaced points, since it is connected with the calculation of integrals having variable limits, where the integrand contains a derivative of the external magnetic field.

A form of general solution of the problem which is convenient for applications is chosen in the present report.

First, since in practice the magnetic field propagates in the channel in some finite interval, in place of one channel section with an external magnetic field we consider a system of such sections with the condition that their interaction can be neglected. Weakening of the interaction is achieved by increasing the period of the system, where the wider the range of magnetic Reynolds numbers in which the channel characteristics must be calculated, the larger should be the period of the system. In a mathematical respect the introduction of a system of sections containing magnetic fields leads to the possibility of representing the field quantities in the form of a Fourier series with respect to the coordinate along the channel.

Second, since the external magnetic field is usually assigned on the basis of measurements at evenly spaced points along a segment of some length, it is conveniently approximated by a finite Fourier series, since if the number of terms of the series is equal to the number of measurement points, then the values of the approximating function at these points coincide with the measured values of the field and there is no sense in taking more terms.

Since the Fourier series for the external magnetic field is finite, the Fourier series for the potential is also finite and contains just as many terms. Thus, the realization of these ideas leads to considerable simplification of the form of the solution of the problem in comparison with those of [1, 2]. For the problem

$$\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}-\mathrm{Re}_m\frac{\partial\varphi}{\partial z}=0,$$
$$\varphi|_{y=0}=0,\quad \left.\frac{\partial\varphi}{\partial y}\right|_{y=1}=f,\quad \varphi(z+2L)=\varphi(z)$$

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we obtain the distribution of the dimensionless electric potential in the form

$$\varphi(y, z) = f_0 y + 2\,\mathrm{Real}\sum_{n=1}^{N} f_n \frac{\operatorname{sh}\varkappa_n y}{\varkappa_n \operatorname{ch}\varkappa_n}\exp(-i\alpha_n z), \tag{1}$$
$$\alpha_n = \pi n/L,\ \varkappa_n = \sqrt{\alpha_n(\alpha_n - i\mathrm{Re}_m)},$$

where f_n are the coefficients of the expansion of the dimensionless induction $f(z)$ of the external magnetic field in a finite Fourier series. Half the distance between the conducting walls is taken as the unit of length.

Equation (1) allows one to calculate the sensitivity $S = \varphi(1, 0)$ of the flowmeter for any form of external magnetic field and an arbitrary $\mathrm{Re_m}$. However, a comparison of the results of a calculation by this equation with experimental data showed a greater nonlinearity of the calculated characteristic $S(\mathrm{Re_m})$ than occurs in reality. Such a difference cannot be explained by the different form of the solution in comparison with those of [1, 2], since the statement of the problem remained as before. Consequently, the statement of the problem is itself unsatisfactory. In the usual statement of the problem, the difference in the value of the magnetic field for the fluxes flowing between the pole pieces of the external magnet and outside them is erased. In the present report an attempt is made to approximately allow for this difference, with the following assumptions being used:

1) Qualitatively, the same difference in the value of the magnetic field fluxes in the channel can be obtained with infinitely long pole pieces but with a relative magnetic permeability $\mu(z)$ which varies along the channel;

2) if we assume that the nonuniformity of the induction $f(z)$ of the external magnetic field is due not to nonuniformity of the magnetic field strength but to nonuniformity of the relative magnetic permeability then the function $\mu(z)$ coincides with $f(z)$. With allowance for this we write the equation for the electric potential in the form

$$\partial^2\varphi/\partial y^2 + \partial^2\varphi/\partial z^2 - \mathrm{Re}_m f(z)\partial\varphi/\partial z = 0.$$

The boundary conditions remain the same. For the harmonic of order n we obtain the homogeneous system of differential equations

$$\frac{d^2\varphi_n}{dy^2} - \varkappa_n^2\varphi_n = -i\,\mathrm{Re}_m\sum_{k\neq n} f_{|n-k|}\alpha_k\varphi_k,$$

where

$$\varkappa_n^2 = \alpha_n^2 - if_0\,\mathrm{Re}_m\alpha_n,\ f_l = 0 \quad \text{for} \quad |l| > N.$$

Since we must find $\varphi_n(y)$ in a form convenient for calculations, with allowance for the boundary conditions we transform the differential equations into integral equations:

$$\varphi_n(y) = f_n R_n(y) + i\mathrm{Re}_m\sum_{k\neq n} f_{|n-k|}\alpha_k\int_0^1 K_n(y, \eta)\,\varphi_k(\eta)\,d\eta,$$
$$R_n(y) = \begin{cases} \dfrac{\operatorname{sh}\varkappa_n y}{\varkappa_n \operatorname{ch}\varkappa_n}, & n \neq 0 \\ y, & n = 0, \end{cases} \tag{2}$$
$$K_n(y, \eta) = \begin{cases} \dfrac{\operatorname{sh}\varkappa_n y \operatorname{ch}\varkappa_n(1-\eta) - \operatorname{sh}\varkappa_n(y-\eta)\operatorname{ch}\varkappa_n U(y-\eta)}{\varkappa_n \operatorname{ch}\varkappa_n}, & n \neq 0 \\ y - (y-\eta)\,U(y-\eta), & n = 0, \end{cases}$$

where U(y) is a unit step function [3]. We solve the system of integral equations (2) approximately using the method of averaging of functional corrections [4]. We introduce the averaging operation with respect to the y coordinate:

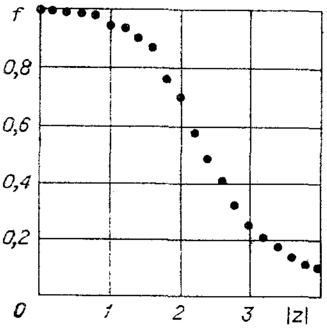

Fig. 1

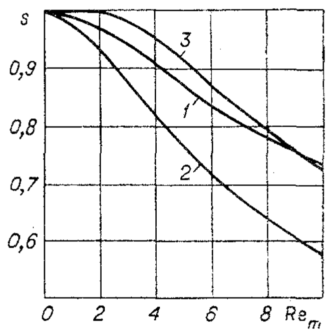

Fig. 2

$$\langle \Psi \rangle = \int_0^1 \Psi(y)\, dy.$$

Using this operation the solution can be represented in the form

$$\varphi_n(y) \approx f_n \frac{R_n(y)\langle T_n \rangle - \langle R_n \rangle T_n(y)}{\langle T_n \rangle} + C_n \frac{T_n(y)}{\langle T_n \rangle}, \tag{3}$$

where

$$T_n(y) = \int_0^1 K_n(y, \eta)\, d\eta,$$

while the coefficients C_n are determined from the system of linear algebraic equations

$$\sum_{k=-N}^{N} [\delta_{nk} - i\,\mathrm{Re}_m (1 - \delta_{nk}) f_{|n-k|} \alpha_k \langle T_n \rangle] C_k = f_{|n|} \langle R_n \rangle.$$

On the basis of (3) we made calculations of the sensitivity of a flowmeter based on models with a constant and a variable relative magnetic permeability of the channel. As the initial data in the calculations we used the results of measurements of the external magnetic field at evenly spaced points along a segment whose length comprises four channel diameters (Fig. 1). The values of the field at the edges of the segment are an order of magnitude less than the maximum value. The period of the system of fields equals 20 channel diameters. It was assumed that outside the zone where the values of the field were measured the field dies out by an exponential law. The exponent was determined from the values of the field within the zone near its edges. Within the limits of one period of the system the total number of points at which the external field is assigned was 200. In the calculations the external field was approximated by a finite Fourier series with 40 terms, with the root-mean-square deviation from the assigned values of the external field being 1.7%.

With the chosen value of the period of the system of fields one can calculate the potential distribution in the channel for $\mathrm{Re}_m \leqslant 10$ without allowance for the finiteness of the pole pieces and for $\mathrm{Re}_m \leqslant 100$ with allowance for it. In the latter case, the "carry-over" of the potential beyond the limits of the zone of the external magnetic field is relatively small.

In Fig. 2 the results of the calculations are compared with experimental data obtained by workers of the Tallin Factory of Measuring Instruments on a DU-500 installation [5] [curve 1) calculation with $\mu(z) = f(z)$; 2) calculation with $\mu(z) = 1$; 3) experiment].

As follows from Fig. 2, the proposed method of solving the problem of the potential distribution with allowance for the effect of the pole pieces on the electromagnetic processes in the channel allows one to obtain agreement between the results of the calculation and experiment with an accuracy of 5% in the interval of Re_m = 0-10. Thus, the given method can serve as the basis in a calculation of the sensitivity of induction flowmeters at large magnetic Reynolds numbers.

Additional experimental studies with different configurations of the external magnetic field are needed for a final conclusion concerning the applicability of the modified equation for the electric potential.

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PROCESS OF OPENING OF AN INELASTIC DIAPHRAGM IN A SHOCK TUBE

V. A. Volkov, V. N. Parmon, and B. K. Tkachenko

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The problems connected with the mechanism of opening of diaphragms in shock tubes are discussed in [1-3], where a "hinge" model is analyzed. It is usually assumed that after breaking along the incisions the leaves of the diaphragm do not undergo deformations and only perform rotational motion about the point of attachment to the tube; the resistance of the diaphragm at the points of attachment is not taken into account. To calculate the process of opening of a diaphragm we used the model of a so-called freely linked chain, where it is assumed that the pressure forces on the diaphragm are much larger than the elastic forces of the material and the elastic forces can be neglected. Such a situation can arise in explosive and electric-discharge shock tubes, shock wind tunnels, etc.

Photography of the process of opening of a copper diaphragm 1.5 mm thick and 50 mm in diameter at the end of an electric-discharge shock tube [4] was carried out in the present work. An SFR camera was used in the mode of framewise photography with a frequency of $5 \cdot 10^5$ frame/sec. Diaphragms of such a type with a cross-shaped incision 1 mm deep withstood pressures of up to 90 atm. Before the discharge the chamber with a volume of 200 cm^3 was filled with helium to 10 atm and within it occurred the discharge of a battery of capacitors at a voltage of 5.5 kV with a total energy of 30 kJ. According to the estimates of [4], the pressure in the chamber increased to ∿400 atm.

Sequential photographs of the opening process every 8 μsec (exposure time 2.5 μsec) are shown in Fig. 1. It is noteworthy that almost from the moment of breakage along the incisions the open section has a strictly cross-shaped form and retains it until full opening. It was also established experimentally that the leaves are elongated by about 1.5 times owing to their elongation in the process of motion.

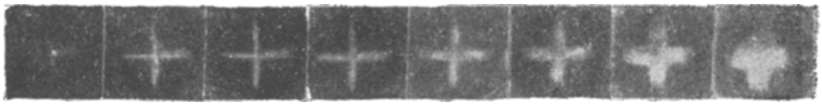

Fig. 1

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